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RAN-2103000206020032**T.Y.B.Sc. (Sem. VI) Examination April - 2023****MTH - 602 - Linear Algebra - II****Time: 2 Hours]****[Total Marks: 50****सूचना : / Instructions**

(१)

नीचे दशविवेक निशानीवाणी विगतो उत्तरवली पर अवश्य लपववी.
Fill up strictly the details of signs on your answer book

Name of the Examination:

T.Y.B.Sc. (Sem. VI)

Name of the Subject :

MTH - 602 - Linear Algebra - II

Subject Code No.: 2103000206020032

Seat No.:

Student's Signature

- (2) All questions are compulsory.
(3) Figures to the right indicate marks of the questions.
(4) Follow usual notations.

Q. 1 Answer the following questions. (Any Five)**(10)**

- (1) State existence theorem of linear transformation $T : U \rightarrow V$.
- (2) Obtain the general rule for the linear transformation $T : V_2 \rightarrow V_2$ defined by $T(0,1) = (1,1)$, $T(1,1) = (2,2)$.
- (3) A linear transformation $T : V_3 \rightarrow V_3$ define by $T(x, y, z) = (x, y, 0)$.
Is T a non singular ? Justify your answer.
- (4) Let $T : V_4 \rightarrow V_7$ be a linear transformation. Is $R(T)$ be a 5th dimensional subspace of V_7 ? Justify your answer.
- (5) Prove that a linear transformation $T : V_3 \rightarrow V_3$ is 1-1 then $r(T) = 3$.
- (6) Is a linear transformation $T : V_3 \rightarrow V_3$ defined by $T(x, y, z) = (x - y, y, z)$ onto ? Justify your answer.
- (7) Find the range and rank of the matrix $\begin{bmatrix} 3 & 5 & 8 \\ 4 & 6 & 10 \end{bmatrix}$.
- (8) Orthonormalized the L.I set $\{(1,0), (1,1)\}$ by Gram Schmidt's process.

Q. 2 Answer the following. (Any two) (10)

- (1) Prove that the set $\{u \in U/T(u) = \theta_V\}$ of linear transformation $T : U \rightarrow V$ be a subspace of U .
- (2) Let $T : U \rightarrow V$ be a linear map. If $u_1, u_2, u_3, \dots, u_n$ are linearly independent vectors of U with $N(T) = \{\theta_U\}$ then prove that $T(u_1), T(u_2), T(u_3), \dots, T(u_n)$ are L.I.
- (3) Obtain the general rule of the linear transformation $T : V_3 \rightarrow V_3$ such that $T(0, 1, 2) = (3, 1, 2), T(1, 1, 1) = (2, 2, 2), T(1, 0, 1) = (1, 0, 4)$.

Q. 3 Answer the following. (Any two) (10)

- (1) Let $T : V_3 \rightarrow V_3$ defined by $T(e_1) = e_1 - e_2, T(e_2) = 2e_2 + e_3, T(e_3) = e_1 + e_2 + e_3$. Prove that T is neither 1-1 nor on-to.
- (2) Is the linear transformation $S : V_3 \rightarrow V_3$ defined by $S(e_1) = e_2 - e_2 + e_3, S(e_2) = 3e_1 - 5e_3$ and $S(e_3) = 3e_1 - 2e_3$ non singular? Find its inverse if exists.
- (3) Verify Rank-Nullity theorem for a linear transformation $T : V_3 \rightarrow V_3$ such that $T(e_1) = e_1 - e_2, T(e_2) = e_2, T(e_3) = e_1 + e_2 - 7e_3$.

Q. 4 Answer the following. (Any two) (10)

- (1) Find the matrices $(T ; B_1, B_2)$ associated with a linear transformation $T : V_2 \rightarrow V_2$ defined by $T(X, Y) = (X, -Y)$ relative to basis $B_1 = \{(1, 1), (-1, 1)\}, B_2 = \{(1, 2), (-2, 1)\}$.
- (2) Find the Range, Rank, Kernel and Nullity of a matrix $\begin{bmatrix} 2 & 0 & 1 \\ 7 & 1 & 2 \\ 3 & -1 & 1 \end{bmatrix}$
- (3) Find the Linear transformation T associated with a matrix $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$ relative to basis $B_2 = \{(1, 0), (2, -1)\}$ and $B_1 = \{(1, 1, 1), (1, 2, 3), (1, 0, 0)\}$.

Q. 5 Answer the following. (Any two)

(10)

- (1) (a) In an Inner Product space V , Prove that $\|\alpha v\| = |\alpha| \|v\|, \forall v \in V,$
 α a scalar
- (b) Explain Euclidean and Unitary inner product space.
- (2) Prove that the set of non zero vectors $\{u_1, u_2, u_3, \dots, u_n\}$ with condition that $u_i \cdot u_j = 0, i \neq j$ and $u_i \cdot u_j \neq 0, i = j$ is linearly independent in an inner product space V .
- (3) Orthonormalized the L.I set $\{(1, 1, 1), (1, -1, 1), (0, 0, 1)\}$ by Gram Schmidt's process.
